- ❖ CARTESIAN SYSTEM & STRAIGHT LINES
- ❖ PAIR OF STRAIGHT LINES & FAMILY OF LINES
- ❖ CIRCLE
- ❖ PARABOLA
- ❖ ELLIPSE
- ❖ HYPERBOLA

Cartesian System and Straight line

1. Distance Between two points in a Plane:

If A(x_1, y_1) and B (x_2, y_2) be two points in a plane, the distance between them is :

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance of O (0,0) from P (*x*,*y*) is $|OP| = \sqrt{x^2 + y^2}$

2. Point of Division on a line Segment:

(a) Internal Division.

Let [AB] be the ine segment joining two points A(x_1, y_1) and B (x_2, y_2) and let P (x, y) be any point on [AB] between A and B such that $\frac{AP}{PB} = \frac{m_1}{m_2}$. then :

$$x = \frac{m_1x_2 + m_2x_1}{m_1 - m_2}, y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} \quad , m_1 + m_2 \neq 0.$$

(b) External Division.

If P divides the line segment [AB] externally in the ratio $m_1 : m_2$, then its co-ordinates are :

$$x = \frac{m_1x_2 - m_2x_1}{m_1 - m_2}, y = \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \quad m_1 \neq m_2$$

Cor . Mid – point Formula. The co-ordinates of the mid-point of the join of (x_1, y_1) and (x_2 , y_2) are :

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

3. Centroid –

If (x_1, y_1) , (x_2, y_2) and (x_3, y_3) be the vertices of a ΔABC , then the co- ordinates of the centroid of ΔABC

are:

$$\left(\frac{x_1 + x_2 + x_3}{2}, \frac{y_1 + y_2 + y_3}{2}\right).$$

4. Incentre -

If $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ be the vertices A, B ,C respectively of ΔABC , with sides BC , CA and AB as a , b, c, then the co-ordinates of the incentre of ΔABC are:

$$\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right).$$

5. Slope

The slope (or gradient) of the line is the tangent of the angle, which the line makes with the positive direction of *x*-axis. The slope of the line is generally denoted by *m*.

If $A(x_1\ ,y_1), B(x_2, y_2)$ are two point of a line l, not parallel to y- axis , then the slope of l. denoted by m, is the ratio $= \frac{y_2-y_1}{x_2-x_1}$, $x_1 \neq x_2$.

Two line (not parallel to y- axis)with slopes m_1 and m_2 are *perpendicular* iff $m_1 m_2 = 1$.

Two line (not parallel to y- axis) are parallel iff their slopes are equal, $i.e. m_1 = m_2$.

6. ANGLE BETWEEN TWO INTERSECTING LINES

The positive angle Θ between lines l_1 and l_2 with slopes $m_1 = \tan\alpha_1 , m_2 = \tan\alpha_2$ respectively is given by:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|.$$

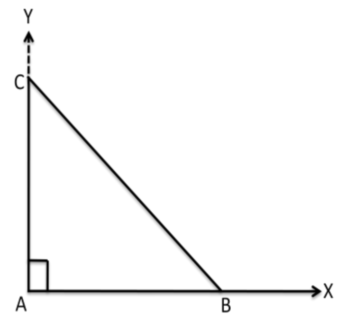
7. AREA OF A TRIANGLE

The area of a triangle with vertices $P(x_1, y_1), Q(x_2, y_2)$ and R(x_3, y_3) is given by :

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\ |(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)|.$$

8. CHOICE OF AXES

Sometimes geometrical problems can be made simple with proper choice Of axes. For the proper choice of axes. we follow the rules as given below:



Rule I. whenever a right-angled triangle is given in a problem, take these lines as co-ordinates axes.

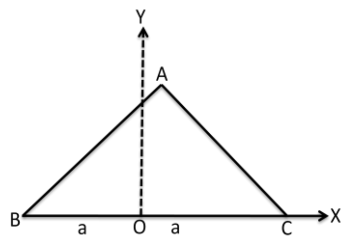
For Ex. Whenever a right-angled triangle is given in a problem, take two lines containing the right angle as co-ordinate axes.

Rules II. Whenever the fixed points A and B are given in a problem, we take mid – point O of [AB] as origin and AB as x —axis and a line through O and perpendicular to AB as $y -$ axis. Thus if |AB|=2a **,** then B is (a,0) and A is (-a,0) .

For Ex. Whenever a ΔABC is given, we take mid-point O of the base [BC] as origin and base BC as $x -$ axis and line through O and perpendicular to BC as $y -$ axis. Thus if |BC| =2 a , then C is $(a, 0)$ and B is $(-a, 0)$.

9. LOCUS

Definition. The locus of a point is the path traced out by the point under certain geometriacal condition/conditions.



Equation of locus is an equation in x and y, which is satisfied by the co-ordiantes of any point on the locus and by the co-ordinates of no other point.

To obtain the equation of the locus:

(i) Let P(x, y) be any point on the locus.

(ii)Apply the given condition/conditions along with a frame, if possible.

(iii) Eliminate the unknown , if any and simplify.

KEY POINT

$Equation\ of\ x -$ axis is $y = 0$

$Equation\ of\ y -$ axis is $x = 0$

10. FORMS OF EQUATIONS OF STRAIGHT LINE

(a) lines Parallel to Axes

equation of straight line parallel to *x-axis* at a distance 'a' is $y = a$ and equation of straight line parallel to $y - axis$ at a distance 'b' is $x = b$.

(b) point-slpe form

The equation of the straight line passing through two points P(x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.

(c) Two-point Form

The equation of the straight line passing through two point P(x_1, y_1) and Q (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ or } \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1} .$$

(d) Slope-Intercept Form

The equation of the straight line, which cuts off a given intercept c on $y -$ axis and makes an angle Θ with $x -$ axis is :

$$y = mx + c, \qquad \text{where } m = \tan\theta.$$

(e) Intercept Form

The equation of a straight line making intercepts 'a' and 'b' on $x -$ axis and $y -$ axis respetively is $\frac{x}{a} + \frac{y}{b} = 1$.

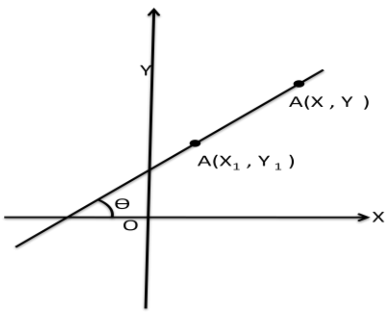
(f) General Form

The equation $ax + by + c = 0$ always represents a strainght line, provided A and B aer not both zero simultaneously.

(g) Symmetric Form

The equation of the st. lne through (x_1, y_1) and inclined at an angle Θ with pos direction of *x*-axis in symmetrical form is :

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r.$$



(h) Normal Form

The equation of the line *l* in terms of *p*, the length of perpendicular from origin on line and angle α which this perpendicular makes with *x*-axis is $x\cos\alpha + y\sin\alpha = p$.

11. DISTANCE OF A POINT FROM A LINE

Length of perpendicular segment drawn from given point (x_1, y_1) to $ax + by + c = 0$ is :

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} .$$

12. POINT OF INTERSECTION

The point of intersection of strainght lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is:

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_2}{a_1b_2 - a_2b_1}\right) \text{, where } a_1b_2 - a_2b_1 \neq 0 .$$

13. CHANGE OF AXES :

(I) Translation of Axes . If we shift the origin without rotation of axes, we call it is *“ Translation of axes”.*

If we shift the origin to (*h*,*k*) , then :

$x = X + h$ and $y = Y + k$

or equaivalently $X = x - h$ and $Y = y - k$,

where (X,Y) and (*x*,*y*) are current and old co-ordinates respectively.

(ii) Ratation of Axes. if we rotate the axes throught an angle ‘Θ’ in the anti-clockwise direction without changing the origin , we call it is “*Rotation of axes.”*

If we rotate the axes thro’ an angle Θ, then:

$x = X\cos\theta - Y\sin\theta$ and $y = X\sin\theta + Y\cos\theta$

or equivalently, $X = x\cos\theta + y\sin\theta$ and $\text{Y} = y\cos\theta - x\sin\theta$,

where (X,Y) and (*x*,*y*) are current and old co-ordinates respectively.

14. MORE RESULTS

(a) Relative positions of points (X_1 , Y_1) and (X_2 ,Y_2) w.r.t the st. line ax+by+c = 0 .

The points (x_1, y_1) and (x_2, y_2) lie on the same or oppositive sides of the st. line $ax + by + c = 0$ according as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same sign or opposite signs.

(b) Risecot of the angles between two lines.

The equations of the bisectors of the angles between the st. lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, $(a_1b_2 \neq a_2b_1)$ are given by :

$$\frac{a_1x + b_1y + c_1}{\sqrt{a^2{}_1 + b^2{}_1}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a^2{}_2 + b^2{}_2}} .$$

(c) Bisector of the angle containing the origin and that of not containing the origin.

We write given equations as $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$, $c_1, c_2 > 0$ then :

$$\frac{a_1x + b_1y + c_1}{\sqrt{{a_1}^2 + {b_1}^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{{a_2}^2 + {b_2}^2}}$$ is the bisector of the angle containing the origin

and $$\frac{a_1x + b_1y + c_1}{\sqrt{{a_1}^2 + {b_1}^2}} = -\frac{a_2x + b_2y + c_2}{\sqrt{{a_2}^2 + {b_2}^2}}$$ is the bisector of the angle not containing the orging.

PAIR OF STRAIGHT LINES &FAMILY OF LINES

FAMILY OF LINES

1. FAMILY OF LINES

We know that two conditions are required in order to determine a line uniquely. Lines satisfying one condition depend on single essential constant. Such a system of lines is called a *one-parameter* family of lines and the undetermined constant is called the *parameter*.

2.EQUATION OF FAMILY OF LINES

The equation of the fimily of lines through the point of intersection of two lines:

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

is $$a_1x + b_1y + c_1 + k(a_2x + b_2y + c_2) = 0$$

For all values of k.

PAIR OF LINES

3. HOMOGENEOUS EQUATION

The Equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of st. lines passing through the origin if $h^2 \geq ab$.

4. ANGLE FORMULA

The angle between the two lines given by $ax^2 + 2hxy + by^2 = 0$

$$\theta = \tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{a + b}\right).$$

KEY POINT
$ax^2 + 2hxy + by^2$ represents: (i) two coincident or parallel lines if $h^2 = ab$ (ii) two perpendicular lines if $a + b = 0$.

5. EQUATION OF BISECTORS

The equation of the bisectors of the angles between the lines $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}.$$

6. GENERAL EQUATION

The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of lines if

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0.$

Assuming that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two straight lines, then the angel between them is $\theta = \tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{a + b}\right)$.

The lines represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are perpendicular iff $a + b = 0$.

7. EQUATION OF STRAIGHT LINES

Joining the origin to the points of intersection of (I) two lines (II) one line and a curve (III) two curves:

Make one equation homogeneous with the help of the other equation.

CIRCLES

1. DEFINITION OF A CIRCLE

The locus of a point, which moves in a planes, such that its distance from a fixed point in the plane is always constant, is called a circle.

The fixed is the **centre** and the constant distance is called the **radius** of the circle.

2. EQUATION OF CIRCLES

(I) The equation of the circle with centre (h,k) and radius *'r'* is :

$$(x-h)^2+(y-k)^2=r^2$$

(II)The equation of the circle with centre (0 ,0) and radius 'r' is :

$$x^2+y^2=r^2$$

(III) The equation $x^2+y^2+2gx+2fy+c=0$, where g , f and c are constants, represents a circle.

(i) its centre is (-g , -f).

(ii) its radius is $\sqrt{g^2+f^2-c^2}$ where $g^2+f^2-c\geq 0$.

(iii) Length of intercept made by the circle on $x-$ axis is $2\sqrt{g^2-c}$, where $g^2-c\geq 0$ and on y- axis is $2\sqrt{f^2-c}$, where $f^2-c\geq 0$.

(IV) The general equation of second degree in x , y :

$$ax^2+2hxy+by^2+2gx+c=0$$

represents a circle if and only if :

(i) $a=b\neq 0$ (ii) $h=0$ (iii)$g^2+f^2-ac\geq 0$

(V) The equation of the circle when end-points of a diameter are A (x_1,y_1) and B (x_2,y_2) is :

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

(VI) Equation of the circle concentric with $x_2+y_2+2gc+2fy+c=0$ is :

$$x^2+y^2+2gx+2fy+\lambda=0$$

3. PARAMETRIC EQUATIONS

If 'Θ' is a parameter, where $0\leq\theta<2\pi$, then:

(i) the equations $x = r\cos\theta$, $y = r\sin\theta$ are parametric equations of the circle $x^2 + y^2 = r^2$

(ii) the equations $x = h + r\cos\theta$, $y = k + r\sin\theta$ are parametric equations of the circle

$$(x-h)^2 + (y-k)^2 = r^2$$

4.<u>CONDITION O TANGENCY</u>

The Condition that the st. line $y = mx + c$ may touch the circle $x^2 + y^2 = r^2$ is $c = \pm r\sqrt{1+m^2}$.

The equations of the tangent, in the **slope-form,** are $y = mx \pm r\sqrt{1+m^2}$.

5.<u> TANGENTS AND NORMAL</u>

(I) The equation of the tangent at (x_1, y_1) to the circle:

(i) $x^2 + y^2 = r^2$ is $xx_1 + yy_1 = r^2$

(ii) $x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

(II) The equation of the normal at (x_1, y_1) to the

(i) circle $x^2 + y^2 = r^2$ is $xy_1 - x_1 y = 0$ $i.e. \frac{x}{x_1} = \frac{y}{y_1}$

(ii) circle $x^2 + y^2 + 2gx + 2fy + c$ is $\frac{y - y_1}{y_1 + f} = \frac{x - x_1}{x_1 + g}$

(III) Length of the tangent from $(x_1, y_1)$to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is:

$\sqrt{x_1{}^2 + y_1{}^2 + 2gx_1 + 2fx_1 + c}$.

6. <u>CHORD IN TERMS OF MIDDLE POINT</u>

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, whose midpoint is $(x_1, y_1), is$:

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1{}^2 y_1{}^2 + 2gx_1 + 2fy_1 + c \;\; i.e. T = S_1.$$

7.<u> COMMON TANGENTS</u>

- The equation of the tangents from P (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by :

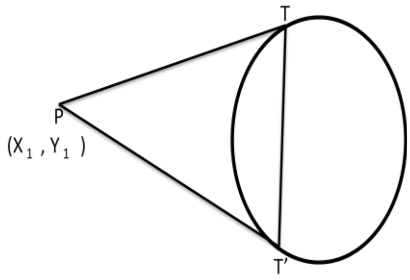
$$(x^2 + y^2 + 2gx + 2fy + c)(\, x_1{}^2 + y_1{}^2 + 2gx_1 + 2fy_1 + c)$$

$$= (xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c)^2$$

$i.e. SS_1 = T^2$

8.<u> CHORD OF CONTACT</u>

If from a point P,PT and PT' are two tangents to the circle, then TT' is the **chord of contant.**



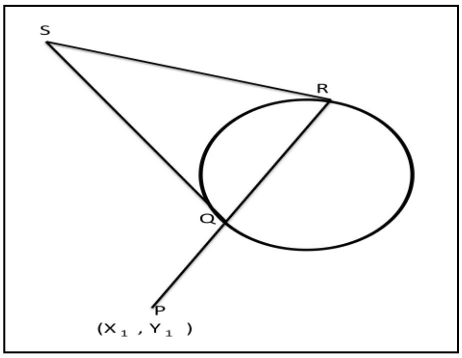
The equation of the chord of contact of tangents drawn from P(x_1, y_1)

to the circle

$(i) x^2 + y^2 = r^2$ is $xx_1 + yy_1 = r^2$

$(ii)\ x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

9. POLE AND POLAR



If P be any point ,Let any secant through P meet the circle at Q and R. The tangents at Q and R meet at S. then the locus of S is called the **polar** of P and P is called the **pole** of the locus of S.

The equation of the polar of P(x_1, y_1) w.r.t the circle:

$(i) x^2 + y^2 = r^2$ is $xx_1 + yy_1 = r^2$

$(ii) x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.$

10. FAMILY OF CIRCLES

(i) **Through line and circle .** The equation of the family of circles passing through the points of intersection of the line $L \equiv lx + my + nx = 0$ and the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is $(x^2 + y^2 + 2gx + 2fy + c)$

$+\lambda(lx + my + n) = 0$, where λ is the parameter.

$(\boldsymbol{ii})$ **Through two circles.** The equation of the family of circles passing through the points of Intersection of two circles.

$$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$$

and $$S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0 \text{ is :}$$

$$(x^2 + y^2 + 2g_1x + 2f_1y + c_1) + \lambda(x^2 + y^2 + 2g_2x + 2f_2y + c_2) = 0$$

where λ is the parameter.

$(\boldsymbol{iii})$ **Through two points.** The equation of the family of cirles passing through two points (x_1, y_1) and (x_2, y_2) is :

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda \begin{vmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0,$$

where λ is the parameter.

$(\boldsymbol{iv})$ **Touching a line.** The equation of the family of circles touching the line at (x_1, y_1) is :

$$(x - x_1)^2 + (y - y_1)^2 + \lambda(ax + by + c) = 0,$$

where λ is the parameter.

$(\boldsymbol{v})$ **Touching both axes.** The equations of the family of circles t\ouching both the axes is :

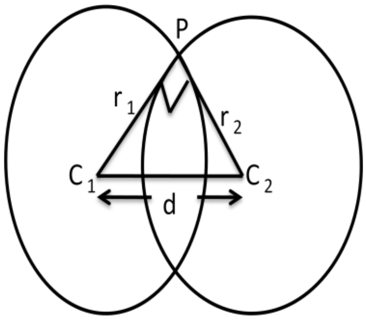
$$x^2 + y^2 \mp 2ax \mp 2ay + a^2 = 0.$$

11. ANGLE BETWEEN TWO CIRCLES

The angle of intersection between two circles whose radii are r_1 and r_2 and d , the distance between their centres is $\cos^{-1}\left(\frac{{r_1}^2+{r_2}^2-d^2}{2r_1r_2}\right)$.

12. ORTHOGONAL CIRCLES

If two circle cut orthogonally (*i.e.* cut at right angles) , then the square of the distance between their centres is equal to the sum of the square of their radii.



$${r_1}^2 + {r_2}^2 = d^2$$

Condition of Orthogonality.

The condition that the two circles.

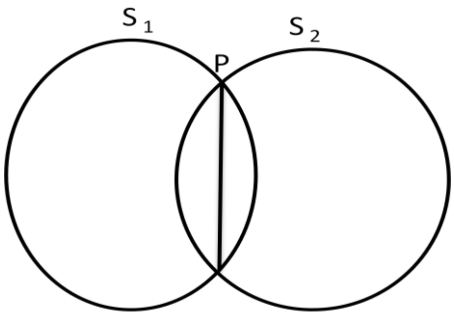
$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$$

and $$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$$

may cut orthogonally is :

$$\boldsymbol{2g_1g_2 + 2f_1f_2 = c_1 + c_2}.$$

13. COMMON CHORD



$$S_1 - S_2 = 0$$

Let $S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1 + c_1 = 0$

and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

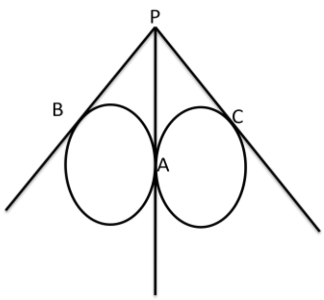
be the intersecting circles.

Then the equation of the common chord is:

$$2(g_1 - g_2)x + 2(f_1 - f_2) + c_1 - c_2 = 0$$

14. RADICAL AXIS –

The radiacal axis of two circles is the locus of a point, which moves in a plane, such that the length of the tangents from it to the two circles are equal.



If $S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

and $S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ be two circle, then the equation of the radical axis is :

$$2(g_1 - g_2)x + 2(f_1 - f_2) + c_1 = 0.$$

The radical axes of the three circles, taken in pairs, meet in a point, which is called as **radical centre.**

15. COAXAL CIRCLES

A *system of circles is said t be coaxal if every pair of which has the same radiacal axis.*

The general equation of the family of co-axal circles is :

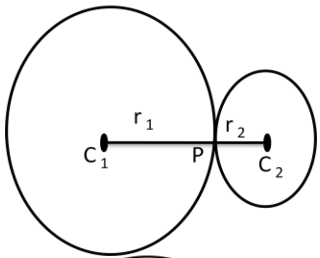
$$x^2 + y^2 + 2gx + c = 0$$

where 'g' varies and c is constant.

16. LIMITING POINTS

The points , which are the centres of the point circles of th co-axal system, are called **limiting points.**

17. TOUCHING CIRCLES



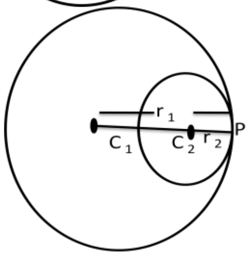
Two circles with centres $C_1(x_1, y_1)$ and $C_2(x_2, y_2)$ and radii r_1 and r_2:

(i)*touch externally* if $|C_1C_2| = r_1 + r_2$

The point of contact is P $\left(\frac{r_1x_2+r_2x_1}{r_1+r_2}, \frac{r_1y_2+r_2y_1}{r_1+r_2}\right)$

(ii) *touch internally* if $|C_1C_2| = |r_1 - r_2|, r_1 \neq r_2$

The point of contanct is P $\left(\frac{r_1x_2-r_2x_1}{r_1-r_2}, \frac{r_1y_2-r_2y_1}{r_1-r_2}\right)$.

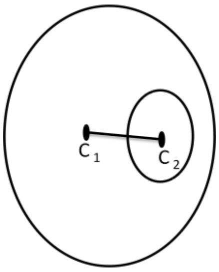


18. COMMON TANGENTS TO THE CIRCLES

Let $S_1 = 0$ and $S_2 = 0$ be two circles with radii r_1 and r_2 and d the distance between their centres.

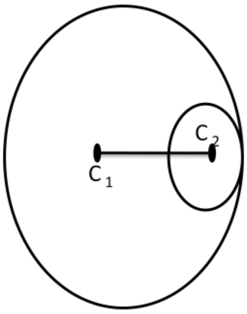
(I) when $r_1 - r_2 > d$, **there is no common tangent**.

Here one circles is completely within the other.



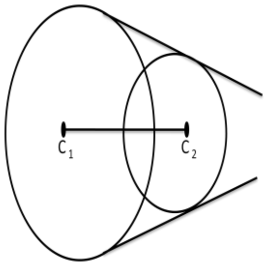
(II) when $r_1 - r_2 = d$, **there is no common tangent**.

Here circles touch each other internally.

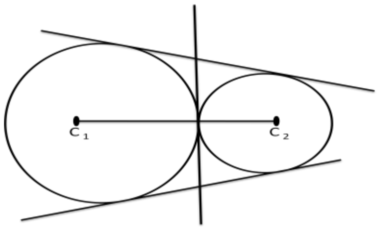


(III) when $r_1 + r_2 > d$ or $r_1 - r_2 < d$, **there are two common tangents**.

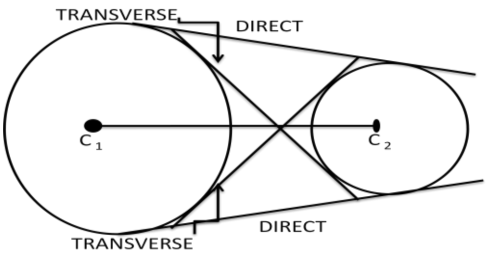
_Here the circles intersect each othere in two distinct points.



(IV) When $r_1 + r_2 = d$, **There are three common tangents**.

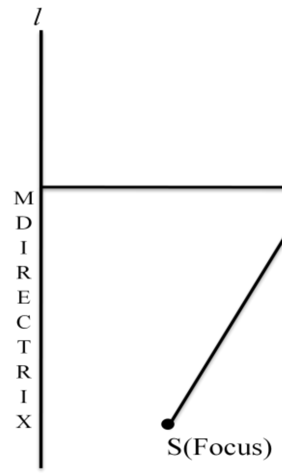


(V) When $r_1 + r_2 < d$, **there are four common tangents**.



PARABOLA

1. DEFINITION



Conic. *A conic is the locus of a point, which moves in a plane, so that its distance from a fixed point is in a constant ratio to its distance from a fixed st. line.*

The fixed point is called **focus,** the costant ratio is called **eccentricity** (e) and fixed st. line is called **directrix.**

Conic is a parabola if $e = 1$, conic is an ellipse if e<1 and conic is hyperbola if e > 1.

2.PARABOLA

The parabola is the locus of a point, which moves in a planes, whose distance from a fixed (called **focus**) equals its distance from a fixed line (called **directrix**)/

3. MAIN FACTS ABOUT PARABOLA

Equation	$y^2 = 4ax$ Right handed	$y^2 = -4ax$ Left handed	$x^2 = 4ay$ Upward	$x^2 = -4ay$ Downward
Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Vertex	(0 , 0)	(0, 0)	(0,0)	(0,0)
Length of latus-rectum	$4a$	$4a$	$4a$	$4a$
Equation latus-rectum	$x = a$	$x = -a$	$y = a$	$y = -a$
Focal distance	$a + x$	$a - x$	$a + y$	$a - y$

4. CONDITION OF INTERSECTION

- The condition that the line $y = mx + c$ may intersect the parabola $y^2 = 4ax$ is $a \geq mc$.

5. CONDITION OF TANGENCY

The condition of tangency in the above parabola is $a = mc$.

6. EQUATIONS OF TANGENTS AND NORMALS

(a) The equation of tangent in slope form and point of contact to the parabola $y^2 = 4ax$

are $y = mx + \frac{a}{m}$, $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$,

(b) Tangent at (x_1, y_1) the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$.

Normal at (x_1, y_1) to the parabola $y^2 = 4ax$ is $y - y_1 = -\frac{y_1}{2a}(x + x_1)$.

7.POSITION OF A POINT

The point (x_1, y_1) lies outside, on, or inside the parabola $y^2 = 4ax$ if ${y_1}^2 - 4ax_1 >, =, < 0$.

8. PARAMETRIC EQUATIONS

Parametic equations of the parabola $y^2 = 4ax$ are $x = at^2$, $y = 2at$, 't' being parameter.

9.EQUATIONS OF TANGENTS AND NORMALS

Tangent and normal at 't' to the parabola $y^2 = 4ax$ are :

$ty = x + at^2$ and $yt + x = 2at + at^2$.

10. CHORD

Chord joining $('t_1', 't_2')$ to parabola $y^2 = 4ax$ is $(t_1, \boldsymbol{t_2})\, y = 2x + 2at_1\boldsymbol{t_2}$.

11. CHORD WITH MID-POINT:

Equation of chord with a given mid-point $(x_1, \boldsymbol{x_2})$ to the parabolic $y^2 = 4ax$ is $yy_{\mathbf{1}} - 2a(x + \boldsymbol{x_1})$

$= y^2{}_{\mathbf{1}} - \boldsymbol{4ax_1}$.

12. MORE RESULTS AND FACTS:

(i) **Chord of Contact.** The equation of the chord of contact of the tangents drawn from (x_1, y_1) to the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$.

(ii) **Polar .** The equation of the polar of the point (x_1, y_1)w.r.t the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$.

(iii) **Pair of Tangents.** The equation of the pair of tangents drawn from (x_1, y_1) to the parabola $y^2 = 4ax$ is

$$(y^2 - 4ax)({y_1}^2 - 4ax_1) = [yy_1 - 2a(x + x_1)]^2.$$

(iv) **Equation of normal in slope form.** The equation of the normal to the parabola $y^2 = 4ax$ in the slope-form is:

$$y = mx - 2am - am^3$$

and the **foot of the normal** is $\boldsymbol{(am^2, -2am)}$.

(v) Diameter. A line , which bisects a system of parallel chords of a parabola is called its **diameter** of the parabola.

(vi) Co- normal Points. The points on the parabola, the normals at which are concurrent, are called co-normal points.

If $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are co-normal points of the parabola $y^2 = 4ax$, then $y_1 + y_2 + y_3 = 0$.

ELLIPSE

1. ELLIPSE

It is the locus of a point, which moves in a plane, so that its distance from a fixed point is in a constanct ratio (less than one) to its distance from a fixed line.

The fixed point is called the focus, the constant ration is called the **eccentricity** (e) and the fixed line is called **directrix.**

Another Difinition

an ellipse is the locus of a point, which moves in a plane, the sum of whose distances from two fixed points is constant. The two fixed points are called **foci.**

2. MAIN FACTS

Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $a>b>0$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ $a>b>0$
Equation of major axis	$y=0$	$x=0$
Equation of minor axis	$x=0$	$y=0$
Length of major axis	$2a$	$2a$
Length of minor axis	$2b$	$2b$
Vertices	$(\pm a,0),(0,\pm b)$	$(\pm b,0),(0,\pm a)$
Foci	$(\pm ae,0)$	$(0,\pm ae)$
Directrices	$x=\pm\frac{a}{e}$	$y=\pm\frac{a}{e}$
Centre	$(0,0)$	$(0,0)$
Equations of latera-recta	$x=\pm ae$	$y=\pm ae$
Length of latus-rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Focal distance of a point (x,y)	$a+ex, a-ex$	$a+ey, a-ey$

3. CONDITION OF INTERSECTION

The condition that the line $y=mx+c$ may intersect the ellipse:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ is } c^2\le a^2m^2+b^2.$$

4. CONDITION OF TANGENCY

The condition that the line $y=mx+c$ may touch the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ is } c^2 = a^2m^2+b^2.$$

5. EQUATION OF TANGENTS AND NORMALS

(a) The equation of the tangent in slope-form and point of contact to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are :

$$y = mx \pm \sqrt{a^2m^2-b^2}\ ;\left(\frac{a^2m}{\sqrt{a^2m^2+b^2}}\ ,\frac{b^2}{\sqrt{a^2m^2+b^2}}\right).$$

(b) The equation of the tangent at (x_1, y_1) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is :

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

and the equation of the normal at (x_1, y_1) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is :

$$\frac{x-x_1}{x_1/a^2}=\frac{y-y_1}{y_1/b^2}\ .$$

6. POSITION OF A POINT

The point (x_1, y_1) lies outside, on or inside the ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if

$$\frac{{x_1}^2}{a^2}+\frac{{y_1}^2}{b^2}-1 >, =, > 0.$$

7. PARAMETRIC EQUATIONS

Parametric equations of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are :

$$x = a\cos\theta\ , y = b\sin\theta\ ;\ \theta \text{ being the parameter.}$$

8. EQUATIONS OF TANGENTS AND NORMALS

Tangent and normal at 'Θ' to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are :

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$$

and

$$ax\sec\theta - by\csc\theta = a^2-b^2.$$

9. MORE RESULTS AND FACTS

(I) Chord of Contact. The equation of the chord of contact of tangents drawn from (x_1, y_1) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is :

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1.$$

(II) Pole and Polar. Let (x_1, y_1) be a point. Then the equation of the polar of (x_1, y_1) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is :

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

The point (x_1, y_1) is the **pole** of (1).

(III) Chord in terms o mid-point . The equation of the chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in terms of its mid-point (x_1, y_1) is :

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1 = \frac{{x_1}^2}{a^2} + \frac{{y_1}^2}{b^2} - 1$$

i.e. $$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{{x_1}^2}{a^2} + \frac{{y_1}^2}{b^2}.$$

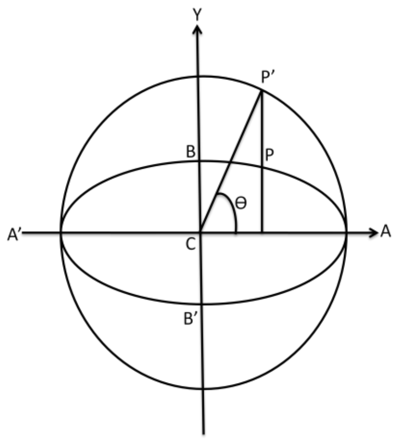
(IV) Pair of Tangents. The equation of the pair of tangents drawn from $(x_1, y_1)$is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is :

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{{x_1}^2}{a^2} + \frac{{y_1}^2}{b^2} - 1\right) = \left(\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1\right)^2.$$

(V) Director Circles. *It is the locus of a point of intersection of perpendicular tangets to the ellipse.*

The director circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is :

$$x^2 + y^2 = a^2 + b^2.$$



(VI) Auxiliary circle. *It is the circle on the major axis the ellipse as diameter.*

The auxiliary circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is :

$$x^2 + y^2 = a^2.$$

Let P be a point on the ellipse and P' is a point on the auxiliary circle such that P' lies on the ordinate (produced) of the point P. Then the co-ordinates of P are (a cos Θ , b sin Θ), where $\theta = \angle ACP'$.

(VII) Diameter. *It is the locus of mid-point of a system of parallel chords of the ellipse.*

The equation of the diameter the ellipse is $y = -\frac{b^2}{a^2 m}x$, where *'m'* is the slope of parallel chords.

HYPERBOLA

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1. HYPERBOLA

It is the locus of a point, which moves in a plane, so that its distance from a fixed point is in a constant ratio (greater than one) to its distance from a fixed line.

the fixed point is called the **focus,** the constant ratio is called the **eccentrity** (e) and the fixed line is called **directrix.**

Another Definition. A *hyperbola is the locus o fa point, which moves in a plane, the difference of whose distances from two fixed points is a positive constant.*

The two fixed points are called **foci.**

2. MAIN FACTS :

Equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $a>0, b>0$	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ $a>0\,, b>0$
Equation of transverse axis	$y=0$	$x=0$
Equation of conjugate axis	$x=0$	$y=0$
Length of transverse axis	$2a$	$2a$
Length of conjugate axis	$2b$	$2b$
Vertices	$(\pm a, 0)$	$(0\,, \pm a)$
Faci	$(\pm ae\,,0)$	$(0\,, \pm ae)$
Directrices	$x=\pm\frac{a}{e}$	$y=\pm\frac{a}{e}$
Centre	(0,0)	(0,0)
Equations of latera-recta	$x=\pm ae$	$y=\pm ae$
Length of latus-rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$
Focal distances of a point (x ,y)	$ex+a\,, ex-a$	$ey+a\,, ey-a$

3.CONDITION OF INTERSECTION:

The condition that the line $y=mx+c$ may intersect the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $c^2\geq a^2m^2-b^2$.

4. CONDITION OF TANGENCY:

The condition that the line $y=mx+c$ may touch the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $c^2=a^2m^2-b^2$.

5.EQUATIONS OF TANGENTS AND NORMALS

(a) The equations of the tangents in slope-form to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are:

$Y=mx\pm\sqrt{a^2m^2-b^2}$; Points of contact are $\left(\frac{a^2m}{\sqrt{a^2m^2-b^2}}\,,\frac{b^2}{\sqrt{a^2m^2-b^2}}\right)$.

(b) The equation of the tangent at (x_1, y_1) to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is :

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

and the equation of the normal at $(x_1, y_1)$to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :

$$\frac{x - x_1}{x_1/a^2} + \frac{y - y_1}{y_1/b^2} = 0.$$

6. <u>POSITION OF A POINT:</u>

The point (x_1, y_1) lies outside, on , or inside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

If $\frac{{x_1}^2}{a^2} - \frac{{y_1}^2}{b^2} - 1 >, =, < 0.$

7. <u>PARAMETRIC EQUATIONS:</u>

Parametric equations of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are:

$x = a\sec\theta \ , y\ b\tan\theta\ ;'\ \theta'$ being the parameter.

8. <u>EQUATION OF TANGENTS AND NORMALS</u>

Tangent and normal 'Θ' to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are:

$\frac{x}{a}\sec\theta + \frac{y}{b}\tan\theta = 1$ and $ax\cos\theta + by\cot\theta = a^2 + b^2$.

9. <u>RECTANGULAR HYPERBOLA:</u>

The hyperbola for which $a = b$ in $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is said to be rectangular hyperbola.

It is also known as **equilateral hyperbola.**

Its equation is $x^2 - y^2 = a^2$ and its eccentricity = $\sqrt{2}$.

10. <u>CONJUGATE HYPERBOLA :</u>

The hyperbola whose transverse and conjugate axis are respectively the conjugate and transverse asix of the given hyperbola is called conjugate hyperbola.

The equation of conjugate hyperbola of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$.

11. <u>ASYMTOTES:</u>

(I) *An asymptote to a curve is a line which touches the curve at infinity but is not wholly at infinity.*

(ii) The equations of the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

$i.e. \frac{x}{a} \pm \frac{y}{b} = 0.$

(iii) The equation of the hyperbola whose axes are asymptotes is **xy = c²** and whose parametric equations are :

$x = ct\ , y = \frac{c}{t}$; 't' being the parameter.

12. <u>MORE RESULTS AND FAC TS :</u>

(I) **Chord of Contanct.** The equation of the chord of contact of tangents drawn from (x_1, y_1) to The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1.$$

(ii) Pole and Polar. Let (x_1, y_1) be a point. Then the equation of the polar of (x_1, y_1)w.r.t the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :

$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

The point (x_1, y_1) is the **pole of (1).**

(iii) Chord in terms of Mid – point. The equation of the chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in terms of its mid-point (x_1, y_1) is :

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1 = \frac{{x_1}^2}{a^2} - \frac{yy_1}{b^2} - 1$$

$i.e.$ $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = \frac{{x_1}^2}{a^2} - \frac{{y_1}^2}{b^2}$.

(iv) Pair of Tangents. The equation of the pair of tangents drawn from (x_1, y_1) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :

$$\left(\frac{x^2}{a^2} - \frac{y^2}{b^2} - 1\right)\left(\frac{{x_1}^2}{a^2} - \frac{{y_1}^2}{b^2} - 1\right) = \left(\frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1\right)^2.$$

(V) Director circle . *It is the locus o a point of intersection of perpendicular tangents to the hyperbola.*

The director circle of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :

$$x^2 + y^2 = a^2 - b^2.$$

(vi) Auxiliary circle. *It is a circle on the transverse axis of the hyperbola as diameter.*

The auxiliary circle of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2$.

(**vii) Conjugate Diameters.** *Two diameters are said to be conjugate when each bisects chords paralle to the other.*

The diameters $y = m_1 x$ and $y = m_2 x$ of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are conjugate if $m_1 m_2 = \frac{b^2}{a^2}$.

properties. ***(i)*** If two diameters are conjugate with respect to a given hyperbola, then they are also conjugate eith respect to conjugare hyperbola.

(ii) If a diameter a hyperbola in real points, then it meets its conjugate hyperbala in imaginary points.

(viii) Properties of Hyperbola. *(i)* The tangent and normal at any point on the hyperbola bisect the angle between the facal radii of that point.

(ii) The locus of the feet of perpendiculars from the faci on any tangent to the hyperbola is the auxiliary circle.

(iii) The product of perpendiculars from the foci on any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is constant and equal to b^2.